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Subjective housing price expectations, falling natural rates, and the optimal inflation target[☆]

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ABSTRACT

U.S. households' housing price expectations deviate systematically from full-information rational expectations: (i) expectations are updated on average too sluggishly, (ii) expectations initially underreact but subsequently overreact to housing price changes, and (iii) households are overly optimistic (pessimistic) about housing price growth when the price-to-rent ratio is high (low). We show that weak forms of housing price growth extrapolation allow to simultaneously replicate the behavior of housing prices and these deviations from rational expectations as an equilibrium outcome. Embedding housing price growth extrapolation into a sticky price model with a lower-bound constraint on nominal interest rates, we show that lower natural rates of interest increase the volatility of housing prices and thereby the volatility of the natural rate of interest. This exacerbates the relevance of the lower bound constraint and causes Ramsey optimal inflation to increase strongly with a decline in the natural rate of interest.

1. Introduction

The large and sustained booms and busts in housing prices in advanced economies are often attributed to households' excessively optimistic or pessimistic beliefs about future housing prices (Piazzesi and Schneider, 2006; Kaplan et al., 2020). This view is supported by a nascent literature that documents puzzling patterns in housing price expectations. Survey measures of expected future housing prices have been found to be influenced by past changes in housing prices, but appear to underreact to these changes. Additionally, they fail to account for the tendency of housing prices to revert to their mean over time (Kuchler and Zafar, 2019; Case et al., 2012; Armona et al., 2019; Ma, 2020).

Documenting deviations of households' expectations about future housing prices from the full-information rational expectations benchmark is interesting, but does in itself not provide insights into the implications of these deviations for housing markets or the design of monetary policy. Understanding these implications requires a structural equilibrium model that jointly captures the

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quantitative patterns of households' deviations from rational expectations and the behavior of housing prices. We construct such an equilibrium model and use it to derive the monetary policy implications of the observed deviations from rational housing price expectations. To the best of our knowledge, this paper is the first to pursue this task. The key monetary policy insight we derive is that the Ramsey optimal inflation target increases much more strongly with a fall in the natural rate of interest than in a setting with rational housing price expectations.[1]

We begin our analysis by comprehensively quantifying the dimensions along which households' housing price expectations deviate from the full-information rational expectations benchmark. We do so using a single data set, so as to establish a coherent set of quantitative targets for our structural equilibrium model with subjective housing price expectations. Our findings reveal three key dimensions in which household expectations deviate from full-information rational expectations. First, expectations about future housing prices exhibit sluggish updating over time. Second, and new to the housing literature, households' expectations about housing price growth covary positively with market valuation, as measured by the price-to-rent ratio, while actual future housing price growth covaries negatively with market valuation. Third, households initially underreact to observed housing price growth, i.e., are too pessimistic in the first few quarters, but later overreact and exhibit—after approximately twelve quarters—overly optimistic expectations.

We then construct and calibrate a simple housing model with optimizing households and Bayesian belief updating, giving rise to extrapolative expectations about housing price growth. The model reproduces—as an equilibrium outcome—the three deviations of households' beliefs from full-information rational expectations mentioned above, as well as important patterns of the behavior of U.S. housing prices, in particular, the large and protracted swings in the price-to-rent ratio over time. Despite its simplicity, the model demonstrates a surprisingly good quantitative fit.

The simple model offers two important economic lessons: (1) Deviations from rational expectations (RE) are key for understanding observed housing price dynamics. In particular, subjective beliefs help substantially to explain the observed volatility in the price-to-rent ratio, volatility in housing price growth, and the strong autocorrelation in housing price growth. In contrast, a setting with rational housing price expectations is unable to jointly match these features of housing prices. (2) The effects of subjective housing price beliefs on equilibrium housing price dynamics are stronger in a low rate environment, i.e., housing prices become more volatile when the natural rate of interest is lower. Consistent with this prediction, several advanced economies, including the United States, have experienced—concurrently with a decline in the natural rate of interest—a considerable rise in housing price volatility, as we document.

We then analyze the monetary policy implications associated with subjective housing price beliefs and the resulting housing price dynamics. To this end, we introduce subjective beliefs that give rise to housing price extrapolation into a New Keynesian model featuring a housing sector and a lower bound constraint on nominal interest rates. Like the simple model, this full general equilibrium model quantitatively replicates the behavior of housing prices and the patterns of deviations from rational housing price beliefs. To allow for a meaningful examination of monetary policy in the presence of subjective beliefs, subjective housing beliefs are introduced in a manner that prevents monetary policy from manipulating household beliefs to its advantage.[2]

We show analytically that the presence of subjective beliefs gives rise to housing price gaps, i.e., to deviations of housing prices from their efficient level, and that these housing price gaps have two important macroeconomic effects. First, a positive housing price gap, i.e., an inefficiently high housing price, makes it optimal for agents to allocate more resources towards housing investment. For a given level of output, this depresses private consumption and thereby real wages, which manifests itself as a negative cost-push term in the Phillips curve. Second, a positive housing price gap also exerts positive pressure on the equilibrium output gap. When policy seeks to keep the output gap stable, it must implement higher (lower) real interest rates than in a setting with rational expectations, whenever the housing price gap is expected to increase (decrease). Since housing price gaps become more volatile when the average natural rate of interest is low, this feature implies that the *volatility of the natural rate increases* when the average level of the natural rate falls.

The fact that a lower average level of the natural rate also implies more natural rate volatility dramatically exacerbates the lower-bound problem for monetary policy.[3] A more restrictive lower bound forces monetary policy to rely more heavily on promising future inflation in order to lower the real interest rate. Consequently, the Ramsey optimal inflation target increases considerably as the average natural rate falls. In our calibrated model, we find that the optimal inflation target increases by approximately one third of a percentage point in response to a one percentage point decline in the natural rate, with the increase being non-linear and becoming stronger for very low levels of the natural rate. In contrast, under rational expectations, the optimal inflation target is nearly invariant to the average level of the natural rate, because the volatility of the natural rate does not increase as its average level falls.

We also investigate the optimal policy response to housing demand shocks. Under rational expectations, neither inflation nor the output gap respond to housing demand shocks. Yet, in a setting where households extrapolate observed housing price growth, housing shocks move housing prices and these housing price movements get amplified by belief revisions. The belief revisions lead to persistent housing price gaps, i.e., deviations of the housing price from its first-best level, to which monetary policy finds it

1 The Ramsey optimal inflation target is the average inflation rate that the Ramsey planner commits to implement.

2 The model also addresses the critique by Barsky et al. (2007) regarding sticky price models with durable goods. Consistent with the data, the model suggests that housing demand responds more strongly to monetary disturbances than non-housing demand, despite housing prices being fully flexible and goods prices being sticky.

3 The lower bound problem for monetary policy arises because nominal interest rates cannot fall (significantly) below zero whenever there is free convertibility of deposits into cash.

optimal to react. Interestingly, however, housing price gaps generate conflicting effects. On the one hand, inefficiently high housing prices generate negative cost-push pressures, which call for a decrease in the policy rate. On the other hand, inefficiently high housing prices trigger a boom in housing investment, exerting upward pressure on the output gap, which calls for an increase in the policy rate. In our calibrated model, the second effect quantitatively dominates. Optimal monetary policy thus 'leans against' housing price movements, with the optimal strength of the reaction depending on the direction of the shock: following a positive housing preference shock, the increase in the interest rate (nominal and real) is more pronounced than the interest rate decrease following a negative housing demand shock. The presence of the lower bound constraint thus strongly attenuates the degree to which monetary policy leans against negative housing demand shocks.[4]

This paper is related to work by Andrade et al. (2019, 2021) who also study how the optimal inflation target depends on the natural rate of interest in a setting with a lower bound constraint. In line with our findings, they show that an increase in the inflation target is a promising approach to deal with the lower bound problem. While their work considers optimized Taylor rules in a medium-scale sticky price model without a housing sector and rational expectations, the present paper studies Ramsey optimal policy in a model featuring a housing sector and subjective housing expectations.

A number of papers consider Ramsey optimal policy in the presence of a lower-bound constraint, but also abstract from housing markets and the presence of subjective beliefs (Eggertsson and Woodford, 2003; Adam and Billi, 2006; Coibion et al., 2012). This literature finds that lower bound episodes tend to be short and infrequent under Ramsey optimal policy, so that average inflation is very close to zero. The present paper shows that this conclusion is substantially altered in the presence of subjective housing price expectations.

Several papers analyze the implications of subjective housing beliefs for housing price dynamics, but do not study monetary policy design (Adam et al., 2012; Glaeser and Nathanson, 2017; Schmitt and Westerhoff, 2019). Optimal monetary policy in the presence of subjective beliefs has previously been analyzed in Caines and Winkler (2021) and Adam and Woodford (2021). These papers abstract from the lower bound constraint and consider different belief setups that are not calibrated to replicate patterns of deviations from rational housing price expectations as observed in survey data. We show that taking into account the existence of a lower bound constraint on nominal interest rates is quantitatively important for understanding how the optimal inflation target responds to lower natural rates.

2. Empirical properties of housing price expectations

In this section, we document three key dimensions in which households' housing price expectations deviate in systematic ways from full-information rational expectations (RE). First, expectations about future housing prices are updated sluggishly. Second, and new to the literature, housing price growth expectations covary positively with the price-to-rent ratio, while actual housing price growth correlates negatively. Third, households' expectations initially underreact but later overreact to observed housing price growth.

2.1. Data

We measure households' expectations about housing price growth using the Michigan household survey. The survey provides subjective expectations about nominal four-quarter-ahead housing price growth, $E_t^{\mathcal{P}}[q_{t+4}/q_t]$, where q_t is the housing price, for the period 2007–2021. We study both mean and median household expectations.[5] The survey also provides housing price growth expectations over the next five years, which we analyze in a robustness exercise.[6]

We measure housing prices using the S&P/Case–Shiller U.S. National Home Price Index and measure housing prices q_t at quarterly frequency by the average of the monthly housing price index. We consider both nominal and real housing prices with real housing prices being obtained by deflating nominal housing prices with the CPI. In Appendix A.1, Table A.1 provides summary statistics of the PR ratio, as well as expected and actual house price growth. Figures A.1 and A.5 plot the respective time series.

2.2. Sluggish updating about expected housing prices

We start by documenting that household expectations about the future level of housing prices are updated too sluggishly. This can be tested following the approach of Coibion and Gorodnichenko (2015), which uses regressions of the form

$$q_{t+4} - E_t^{\mathcal{P}}\left[q_{t+4}\right] = a^{CG} + b^{CG} \cdot \left(E_t^{\mathcal{P}}\left[q_{t+4}\right] - E_{t-1}^{\mathcal{P}}\left[q_{t+3}\right]\right) + \varepsilon_t. \tag{1}$$

[4] In an extension, we analyze whether macroprudential policies could address the housing market inefficiencies resulting from subjective housing beliefs. We find that taxes on housing would have to be very large, very volatile, and often negative, see Online Appendix D.

[5] Analyzing the dynamics of individual expectations over time is difficult because households in the Michigan survey are sampled at most twice. The expected housing price growth rates from the Survey of Consumers are available since 2007 which is why we focus on the period 2007–2021.

[6] The main analysis focuses on the short horizon expectations because these determine housing prices according to our model.

Table 1
Sluggish adjustment of housing price expectations.

	Mean expectations	Median expectations
Nominal housing prices		
$\hat{b}^{CG}$	2.22***	2.85***
	(0.507)	(0.513)
R^2	0.59	0.65
N	52	52
Real housing prices		
$\hat{b}^{CG}$	2.00***	2.47***
	(0.332)	(0.366)
R^2	0.59	0.62
N	52	52

Notes: This table shows the estimates of regression (1) for nominal and real housings prices and using mean and median expectations, respectively. The standard errors in parentheses are robust with respect to heteroskedasticity and autocorrelation (Newey–West with four lags). Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

The regression projects forecast errors about the future housing price level on the change in the four-quarter-ahead expected housing price. Under the full-information RE hypothesis, information that is contained in agents' information set, i.e., past forecasts, should not predict future forecast errors (H_0: $b^{CG} = 0$).[7]

We estimate Eq. (1) for nominal and real housing price growth and using mean and median expectations, respectively. We compute subjective expectations about the future housing price level as $E_t^{\mathcal{P}}\left[q_{t+4}\right] = E_t^{\mathcal{P}}\left[q_{t+4}/q_t\right] q_t$, where $E_t^{\mathcal{P}}\left[q_{t+4}/q_t\right]$ is the housing price growth expectations from the Michigan survey and q_t the S&P/Case–Shiller index. When considering real housing prices, we deflate the nominal housing price growth expectations using the subjective (mean/median) inflation expectations from the Michigan survey.

Table 1 reports the estimated b^{CG} coefficients from regression (1). We find that, inconsistent with the full-information RE hypothesis, the estimated coefficient is positive and statistically significant at the 1% level in all considered specifications. This implies that housing price expectations are updated too sluggishly: when updating their expectations upwards (downwards), households on average underpredict (overpredict) the level of future housing prices. The magnitude of the estimates is also large in economic terms: a coefficient estimate of two suggests that forecast revisions should approximately be three times as strong than they actually are.

2.3. Opposing cyclicality of actual and expected housing price growth

We next document that actual and expected housing price growth covary differently with housing market valuation. In particular, households' expectations about future housing price growth covaries positively with the price-to-rent ratio PR_t, while actual future housing price growth covaries negatively with PR_t. We estimate a pair of regressions of the form

$$E_t^{\mathcal{P}}\left[\frac{q_{t+4}}{q_t}\right] = a + c \cdot PR_{t-1} + u_t \tag{2}$$

$$\frac{q_{t+4}}{q_t} = \mathbf{a} + \mathbf{c} \cdot PR_{t-1} + \mathbf{u}_t, \tag{3}$$

where c captures the covariation of households' expectations about four-quarter-ahead housing price growth with the (lagged) price-to-rent ratio and $\mathbf{c}$ documents the covariation of realized housing price growth with the price-to-rent ratio. Full information rational expectations imply that $c = \mathbf{c}$, whenever the agents' information set includes the past price-to-rent ratio.

Table 2 reports the regression results. Across all specifications, expected housing price growth covaries positively with the price-to-rent ratio, while realized housing price growth covaries negatively: expected housing price growth is thus pro-cyclical, while realized housing price growth is counter-cyclical. This pattern is akin to the one documented in stock markets (Adam et al., 2017). Since the predictor variable used in these regression equations is highly persistent, we compute an estimate for the difference $(\mathbf{c} - c)$ that adjusts for small sample bias (Stambaugh, 1999; Adam et al., 2017). The adjusted estimate turns out to be highly statistically significant in all specifications. Households are therefore overly optimistic (pessimistic) in times of housing booms (busts). This aligns well with findings in Ma (2020), who shows that households expectations fail to incorporate the mean reversion in housing price behavior, so that positive (negative) deviations from fundamental housing prices are associated with above (below) average price growth expectations.

[7] Given that the Survey of Consumers only asks households about their one-year ahead forecasts, i.e., keeps the horizon fixed, the independent variable $E_t^{\mathcal{P}}\left[q_{t+4}\right] - E_{t-1}^{\mathcal{P}}\left[q_{t+3}\right]$ in (1) is not a forecast revision about house prices for the same time period. Under FIRE, however, it would still hold that $b^{CG} = 0$. Furthermore, we show that when we instrument the independent variable with exogenous monetary policy shocks, the results remain robust (see Appendix A.3 for details).

Table 2
Cyclicality of expected vs. actual housing price growth.

	$\hat{c}$ (in %)	$\hat{\mathbf{c}}$ (in %)	bias (in %) $-E(\hat{\mathbf{c}} - \hat{c})$	p-value $H_0 : c = \mathbf{c}$	N
Nominal housing prices					
Mean expectations	0.033	−0.102	0.001	0.000	50
	(0.008)	(0.007)			
R^2	0.015	0.288			
Median expectations	0.014	−0.102	0.009	0.000	50
	(0.001)	(0.007)			
R^2	0.034	0.288			
Real housing prices					
Mean expectations	0.030	−0.113	−0.003	0.000	50
	(0.017)	(0.009)			
R^2	0.005	0.321			
Median expectations	0.010	−0.113	0.006	0.000	50
	(0.004)	(0.009)			
R^2	0.001	0.321			

Notes: $\hat{c}$ is the estimate of c in Eq. (2) and $\hat{\mathbf{c}}$ the estimate of $\mathbf{c}$ in Eq. (3). Standard errors in parentheses are based on Newey–West with four lags. The Stambaugh (1999) small-sample bias correction is reported in the second-to-last column and the last column reports the associated p-values for the null hypothesis $c = \mathbf{c}$, using the small sample bias correction.

Quantitatively, the results imply that a two standard deviation increase of the price-to-rent ratio by 15.5 units increases the mean household expectations about four-quarter-ahead real housing price growth by around 0.5%. Actual four-quarter ahead housing price growth, however, falls by around 1.5%, so that the forecast error is approximately 2%.

2.4. Initial under- and subsequent over-reaction of housing price growth expectations

Finally, we document that households' expectations initially underreact to observed housing price growth but overreact later on. This exercise provides a unified assessment of the previous two findings: While the results in Table 1 show that households update short-term housing price beliefs on average too sluggishly, the results in Table 2 indicate over-optimism when the current market valuation is high, which points to some form of overreaction to past housing price increases. It turns out that both patterns can be jointly understood by considering the dynamic response of actual and expected housing price growth to housing price changes.

We investigate the dynamic response of households' forecast errors about housing price growth in response to realized housing price growth following the approach in Angeletos et al. (2020), who analyze forecast errors about unemployment and inflation. We also consider the dynamic evolution of realized cumulative housing price growth to interpret the behavior of forecast errors in light of actual realizations. We estimate local projections (Jordà, 2005) of the form

$$X_{t+h} = a^h + b^h \frac{q_{t-1}}{q_{t-2}} + u_t^h, \tag{4}$$

where X_{t+h} is either the cumulative capital gain q_{t+h+4}/q_t, or the forecast error about four-quarter-ahead housing price growth $q_{t+h+4}/q_{t+h} - E_{t+h}^{\mathcal{P}}[q_{t+h+4}/q_{t+h}]$. u_t^h is a potentially autocorrelated and heteroskedastic residual term.

Fig. 1(a) shows the dynamic response of cumulative housing price growth to realized housing price growth, as captured by the estimated coefficients b^h. The initial housing price growth is not only persistent, but increases further over time, reaching a plateau after around twelve quarters. This finding is consistent with the high autocorrelation displayed by housing price growth.

Fig. 1(b) shows the dynamic response of forecast errors to realized housing price growth. Forecast errors are initially positive but later on—once cumulative housing price growth reaches its plateau—become negative before eventually disappearing. The positive values in the initial periods indicate that agents' expectations react too sluggishly: realized housing price growth is persistently larger than expected. This implies that initially expectations underreact to the observed change in housing prices. Subsequently, when housing price growth has fully materialized and housing prices plateau, agents are still optimistic about future housing price growth, thereby over-estimating future housing price growth. This is consistent with the fact that housing price growth expectations display the wrong cyclicality with housing market valuation. It also implies that households entirely miss the mean-reversion in housing price growth: forecast errors turn negative once housing prices stop increasing, with the negative forecast errors disappearing only slowly over time. This pattern is consistent with the experimental evidence provided in Armona et al. (2019).

Interestingly, Li et al. (2023) show that professional forecasters expect strong mean reversion in housing price growth, unlike households. Professionals expect, however, mean reversion that is stronger than in the data. Similarly to households, professionals' house price growth expectations covary positively with past house price growth and forecast errors are positively predicted by past forecast revisions at the consensus level.

2.5. Robustness

Our results are robust to modifications of the empirical specifications along a number of dimensions.

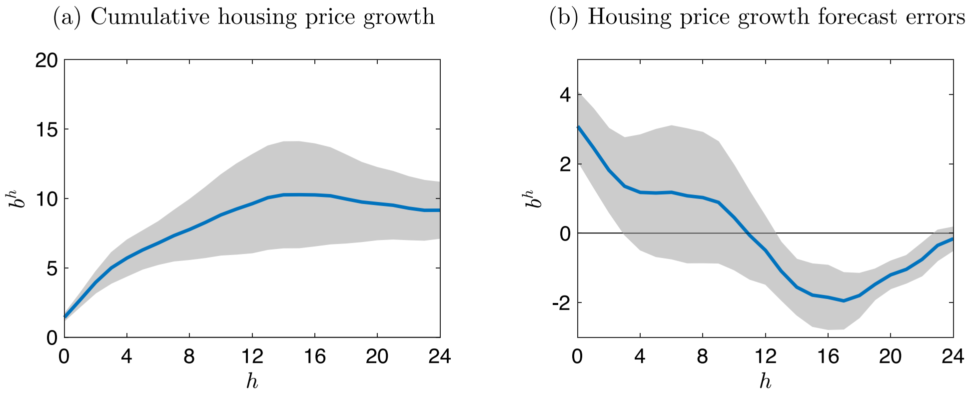

Fig. 1. Dynamic responses to a realized housing price growth. *Notes*: Panel (a) shows the dynamic response of cumulative real housing price growth at horizon h to a one standard deviation innovation in housing price growth. Panel (b) reports the dynamic response of housing price forecast errors at horizon h of one-year ahead expectations to a one standard deviation innovation in housing price growth. Positive (negative) values indicate that realized housing price growth exceeds (falls short of) expected housing price growth. The shaded area shows the 90% confidence intervals, standard errors are robust with respect to autocorrelation and heteroskedasticity (Newey–West with $h+1$ lags).

Five-year ahead expectations. We investigate the robustness of our results by considering different horizons for housing price growth expectations. Appendix A.2 shows that our results in Sections 2.2 and 2.3 are robust to this.[8]

Instrumental-variable estimation. Appendix A.3 shows that our findings in Section 2.2 that households update their expectations sluggishly are robust to using an instrumental-variable approach for estimating regression (1), in which forecast revisions are instrumented with monetary policy shocks obtained via high-frequency identification.

Sluggish updating of housing price growth expectations. Appendix A.4 shows that similar results emerge when using actual and expected housing price growth in Eq. (1) instead of the level and expected level of the housing price.

Cyclicality of housing price forecast errors. Appendix A.5 shows that similar results as in Section 2.3 are obtained when first subtracting Eq. (2) from (3) and estimating the resulting equation with forecast errors on the left-hand side, as in Kohlhas and Walther (2021) who do not consider housing related variables.

Forecast error dynamics with median expectations. In Appendix A.6, we show that the nominal forecast error responses look very similar to the ones for real forecast errors presented in Section 2.4. Likewise, using median expectations instead of mean expectations makes no noticeable difference for the results.

Excluding the COVID-19 period. Appendix A.7 shows that all our results are robust to ending the estimation sample in 2019, thereby excluding the COVID-19 period.

Analysis using regional data. As is well known, housing prices often display considerable regional variation across the United States. We thus check whether the three deviations from full-information RE documented above are also present in regional housing prices and housing price beliefs. Appendix A.8 uses regional housing price indices and exploits local information contained in the Michigan survey that allows grouping survey respondents into different U.S. regions. Repeating the above analyses at the regional level shows that one obtains quantitatively similar results.

Monthly data. In Appendix A.9, we consider monthly data instead of quarterly data and find that our results remain robust to this change.

3. Extrapolative housing price expectations, equilibrium housing prices, and the natural rate of interest

This section presents a stylized housing model in which Bayesian learning causes households to extrapolate past housing price growth into the future. The model makes equilibrium predictions about the joint dynamics of housing prices and housing price beliefs, with housing prices depending on housing price beliefs and housing price beliefs being influenced by past housing price behavior. Despite the simplicity of the model, the equilibrium dynamics quantitatively replicate key features of U.S. housing price behavior and the patterns of deviations from rational expectations documented in Section 2.

The model predicts that large parts of observed housing price volatility are due to the presence of subjective housing price beliefs. In addition, it predicts that low levels of the natural rate of interest give rise to increased housing price volatility. As we show, this prediction is consistent with the evolution of natural rates and housing prices in advanced economies over the past decades.

[8] The sample is too short to obtain significant results for the analysis performed in Section 2.4.

3.1. A simple model with extrapolative housing price beliefs

Consider a unit mass of identical households.[9] Each household chooses consumption $C_t \geq 0$, bonds B_t, housing units to own $D_t \in [0, D^{max}]$, and housing units to rent $D_t^R \geq 0$ to maximize lifetime utility[10]

$$E_t^{\mathcal{P}}\left[\sum_{t=0}^{\infty} \beta^t \left[C_t + \xi_t^d \left(D_t + D_t^R\right)\right]\right]$$

subject to the budget constraint

$$C_t + B_t + \left(D_t - (1-\delta)D_{t-1}\right) q_t + R_t D_t^R = Y_t + (1+r_t)B_{t-1}$$

for all $t \geq 0$, where Y_t is the total endowment (assumed to be sufficiently large), q_t the real price of housing, r_t the real interest rate, R_t the real rental price, and δ the housing depreciation rate. ξ_t^d denotes the preference for housing and is an exogenous determinant of housing prices. Household expectations are based on the subjective probability measure $\mathcal{P}$, as specified below.

The households' optimality condition for owned housing implies that equilibrium housing prices are determined by the asset pricing equation[11]

$$q_t = \xi_t^d + \beta(1-\delta)E_t^{\mathcal{P}}[q_{t+1}] \tag{5}$$

and the optimality condition for rental units gives

$$R_t = \xi_t^d. \tag{6}$$

The optimal consumption-savings decision

$$1 = \beta(1+r_t) \tag{7}$$

implies that the real interest rate r_t is constant and equal to the natural rate of interest $r^* = 1/\beta - 1$ at all times.

We now introduce subjective beliefs that give rise to housing price growth extrapolation, using the setup in Adam et al. (2016). Households perceive housing price growth to evolve according to

$$\frac{q_t}{q_{t-1}} = b_t + \varepsilon_t, \tag{8}$$

where $\varepsilon_t \sim iiN(0, \sigma_\varepsilon^2)$ is a transitory component of housing price growth and b_t a persistent component, which itself evolves according to $b_t = b_{t-1} + \nu_t$, with $\nu_t \sim iiN(0, \sigma_\nu^2)$. Households observe the realized housing price growth (q_t/q_{t-1}) and use Bayesian belief updating to estimate from observed housing price growth the persistent and transitory components. With conjugate prior beliefs, the subjective conditional one-step-ahead housing price growth expectation,

$$\gamma_t \equiv E_t^{\mathcal{P}}\left[\frac{q_{t+1}}{q_t}\right] \tag{9}$$

evolves according to the learning equation

$$\gamma_t = \min\left\{\gamma_{t-1} + \frac{1}{\alpha}\left(\frac{q_{t-1}}{q_{t-2}} - \gamma_{t-1}\right), \overline{\gamma}\right\}, \tag{10}$$

where $1/\alpha$ is the Kalman gain and $\overline{\gamma}$ an upper bound on housing price growth beliefs, which ensures that housing price growth optimism is bounded from above, so as to keep subjectively expected utility finite. The gain parameter $1/\alpha$ captures the degree of extrapolation, i.e., it determines how strongly past housing price growth surprises feed into households' housing price growth beliefs for the future. In Section 3.3, we show that modeling subjective beliefs in this way performs well in matching the empirical findings discussed in Section 2.

From the asset pricing Eq. (5) and the definition of subjective beliefs γ_t it follows that the equilibrium housing price is given by

$$q_t = \frac{1}{1 - \beta(1-\delta)\gamma_t}\xi_t^d. \tag{11}$$

From the rental price (6) it follows that the equilibrium price-to-rent ratio is given by

$$PR_t \equiv \frac{q_t}{R_t} = \frac{1}{1 - \beta(1-\delta)\gamma_t}. \tag{12}$$

Eqs. (10)–(12) jointly characterize the equilibrium dynamics of housing prices, subjective beliefs, and the price-to-rent ratio.

[9] The fact that households are identical is not common knowledge among households.

[10] The constraint $D_t \leq D^{max}$ ensures existence of optimal plans in the presence of subjective housing beliefs. It is chosen such that it will never bind in equilibrium: housing supply D is fixed and satisfies $0 < D < D^{max}$.

[11] For the household, the first-order conditions may hold for some contingencies only with inequality under the subjectively optimal plans, due to the presence of short and long constraints. This explains why rational households can hold price expectations that differ from the discounted sum of future rents, see Adam and Marcet (2011) for details and Adam and Nagel (2023) for related arguments.

3.2. Model calibration

We now calibrate the six parameters describing our simple model. Two parameter values are chosen from the literature and four are set to target data moments.

We estimate the updating gain $1/\alpha$ by estimating the regression

$$\gamma_t - \gamma_{t-1} = \beta_0 + \beta_1 \left(\frac{q_{t-1}}{q_{t-2}} - \gamma_{t-1} \right) + \epsilon_t, \tag{13}$$

where β_1 is our estimate of $1/\alpha$. We use median expectations of house price growth divided by median inflation expectations from the Survey of Consumers as our γ_t.[12] We obtain a point estimate of 0.007 when not including an intercept, and 0.0065 when allowing for an intercept.[13] These values are practically identical to the value of 0.007 estimated in Adam et al. (2016) for stock price growth expectations. We therefore set $1/\alpha = 0.7\%$. The low value for the Kalman gain implies that agents extrapolate observed capital gains only weakly, because households believe most of the realized capital gains to be due to transitory components. The value for the upper belief bound $\overline{\gamma}$ is set to match the maximum observed deviation of the price-to-rent ratio from its mean over the same period. This yields $\overline{\gamma} = 1.0031$.

The annual housing depreciation rate is set equal to 3% following Adam and Woodford (2021). The quarterly discount factor β is set such that the (annualized) natural interest rate is equal to 0.75%, which is the average value of the U.S. natural rate over the period 2007–2021 estimated by Holston et al. (2017).

It only remains to specify the process for housing preference shocks. We consider an AR(1) process

$$\log \xi_t^d = \left(1 - \rho_\xi\right) \log \underline{\xi}^d + \rho_\xi \log \xi_{t-1}^d + \varepsilon_t^d, \tag{14}$$

where $\varepsilon_t^d \sim iiN(0, \sigma_\xi^2)$. To give the rational expectations version of the model a chance to replicate the observed high quarterly persistence of the price-to-rent ratio, we set $\rho_\xi = 0.99$.[14] The standard deviation of ε_t^d is then chosen such that the model replicates the empirical standard deviation of the price-to-rent ratio over the period 2007–2021, expressed in percent deviation from its mean. This yields $\sigma_\xi = 0.67\%$ for the subjective belief model and $\sigma_\xi = 2.24\%$ for the rational expectations version of the model.[15] The latter is more than three times as large because housing prices in the subjective belief model also fluctuate due to changes in the subjective beliefs.

3.3. Match of empirical patterns in housing prices and beliefs

We now show that the simple model with extrapolative housing price beliefs replicates surprisingly well a number of *untargeted* data moments, including the behavior of the price-to-rent ratio, the behavior of housing price growth, and the previously documented deviations of households' expectations from full-information rational expectations (Section 2).

Table 3 shows in the upper panel that the subjective belief model replicates the empirical volatility and autocorrelation of the price-to-rent ratio as well as of housing price growth. While the standard deviation of the price-to-rent ratio is a targeted moment, all other moments are untargeted. The model matches very well the high quarterly autocorrelation of the price-to-rent ratio and the fairly high quarterly autocorrelation of housing price growth. It undershoots somewhat the standard deviation of quarterly housing price growth, illustrating that it features perhaps too little high-frequency variation in prices.[16]

Table 3 and Fig. 2(a) show that the simple model also quantitatively replicates the three key deviations of households' housing price expectations from rational expectations as documented in Section 2. The lower panel of Table 3 illustrates that it matches the sluggish updating about expected housing prices ($b^{CG} > 0$) and the opposing cyclicality of actual and expected housing price growth ($c > 0$ and $\mathbf{c} < 0$). The magnitudes of the coefficients generated by the model closely match the ones obtained using survey data, except that the model underpredicts somewhat the counter-cyclicality of actual housing price growth. Fig. 2(a) shows that the simple model also matches the dynamic response of forecast errors, where model-implied forecast errors are computed as $FE_{t+h}^{model} = \frac{q_{t+4+h}}{q_{t+h}} - \left(\gamma_{t+h}\right)^4$.[17] These results are a unique success of extrapolative beliefs: these model-based statistics would be identically equal to zero for the rational expectations model.

We now use the subjective belief model to understand the quantitative importance of subjective housing beliefs for housing market outcomes. To this end, we consider the calibrated subjective belief model but impose rational housing expectations. The resulting outcomes are reported in the second to last column of the upper panel in Table 3: the standard deviation of the price-to-rent ratio and of housing price growth both decrease by about 70% and the autocorrelation of housing price growth essentially drops to zero. This shows that the majority of the observed fluctuations in the PR ratio in housing price growth is due to the presence of

[12] We transform the one-year ahead expectations into one-quarter ahead expectations as follows: $\gamma_{t+1} = (\gamma_{t+4})^{1/4}$.

[13] When using nominal instead of real house price growth rates and expectations thereof, we estimate the updating gain to be around 0.009.

[14] The subjective belief model can generate housing price persistence solely via the belief dynamics.

[15] We normalize the mean of the housing preference shock process $\underline{\xi}^d$ to 1, but this is irrelevant for the cyclical properties of housing price beliefs and housing prices in which we are interested in here.

[16] This could easily be remedied by adding some i.i.d. shocks, for example to the discount factor β.

[17] We compute the four-quarter-ahead house price growth expectations implied by the model using the approximation: $E[q_{t+h+4}/q_t] = E[e^{\sum_{j=1}^4 ln(q_{t+h+j}/q_{t+h+j})}] \approx e^{\sum_{j=1}^4 ln(E[q_{t+h+j}/q_{t+h+j})]} = (\gamma_{t+h})^4$, which is exact up to small and time-invariant Jensen-inequality terms.

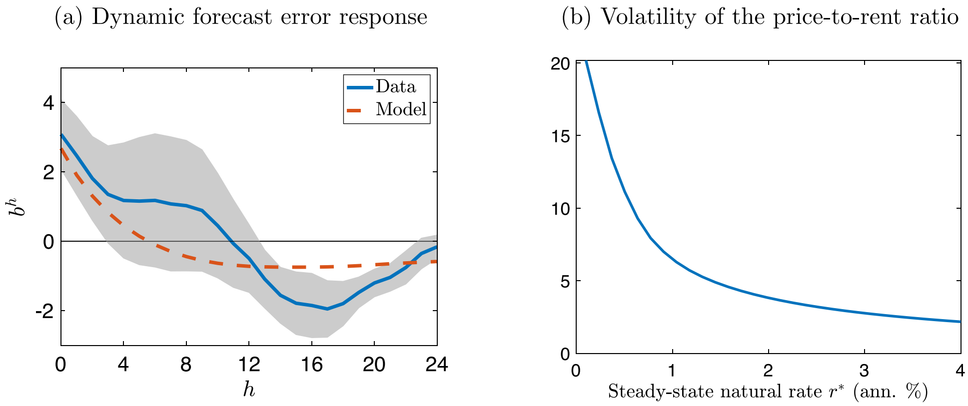

Fig. 2. Properties of the subjective belief model. *Notes*: Panel (a) shows impulse–response functions of housing price forecast errors of one-year ahead expectations to a one standard deviation innovation in housing price growth from the model and the data. The shaded area shows the 90%-confidence intervals of the empirical estimates, standard errors are robust with respect to autocorrelation and heteroskedasticity (Newey–West with $h+1$ lags). Panel (b) reports the standard deviation of the price-to-rent ratio, relative to its mean value, in the subjective belief model as a function of the steady state natural rate of interests.

Table 3
Housing price moments and deviations from RE: data vs. model.

	Data (2007–2021)		Subj. belief model	RE version of subj. beliefs	Recalibrated RE model
Housing price moments					
$\text{Std}\left(PR_t\right)$	8.76		8.76	2.67	8.76
$\text{Corr}(PR_t, PR_{t-1})$	0.99		0.99	0.99	0.99
$\text{Std}(q_t/q_{t-1})$	1.8		1.1	0.3	1.1
$\text{Corr}(q_t/q_{t-1}, q_{t-1}/q_{t-2})$	0.79		0.76	−0.01	−0.02
Deviations from Rational Expectations					
	Mean expect.	Median expect.			
$\hat{b}^{CG}$	2.00	2.47	2.09		
	(0.332)	(0.366)			
$\hat{c}$	0.030	0.010	0.030		
	(0.017)	(0.004)			
$\hat{\mathbf{c}}$	−0.113	−0.113	−0.063		
	(0.009)	(0.009)			

Notes: The upper panel reports the standard deviation and first-order autocorrelation of price-to-rent ratios and housing price growth in the data (2007–2021), for the baseline model with subjective housing price beliefs, the rational expectations version of the subjective belief model (same model parameters) and the recalibrated version of the rational expectations model. The standard deviations are in percentage points. The lower panel reports b^{CG} from regression (1), and c and $\mathbf{c}$ from regressions (2) and (3), respectively (both in %). For the data, we report estimates obtained using mean and median expectations. We do not report the estimates for regressions (1), (2) and (3) for the models with rational expectations.

subjective beliefs. In addition, the autocorrelation of housing price growth is almost exclusively due to the presence of subjective beliefs.

The rational expectations version of the model not only fails to match the observed patterns of deviations from rational expectations, but also has difficulties in matching actual housing price behavior. This is illustrated in the last column of the upper panel in Table 3, which calibrates the rational expectations version of the model in a way that it also matches the standard deviation of the price-to-rent ratio. This requires $\sigma_\xi = 2.24\%$, which is more than three times higher than in the model under subjective beliefs. Perhaps not surprisingly, the rational expectations model has difficulties in generating persistent housing returns: it cannot account for the high autocorrelation in observed housing price growth.

3.4. Implications of falling natural rates for housing price dynamics

This section shows that lower natural rates of interest are associated with higher housing price volatility: this holds true in the data and in the subjective belief model and will be key for understanding the monetary policy outcomes in the next section.

In the subjective belief model, a higher discount factor $\beta < 1$ gives rise to a lower natural rate of interest $r^* = 1/\beta - 1$ (Eq. (7)). A high discount factor also implies that any given change in housing price growth expectations leads to a larger change in equilibrium housing prices (Eq. (11)). Larger realized housing price growth in turn produces stronger revisions in beliefs in the future (Eq. (10)) and thus feeds stronger price growth in the subsequent period. Through this feedback loop, lower natural rates generate stronger momentum and more volatility in beliefs. This leads to more volatility in the price-to-rent ratio (Eq. (12)).

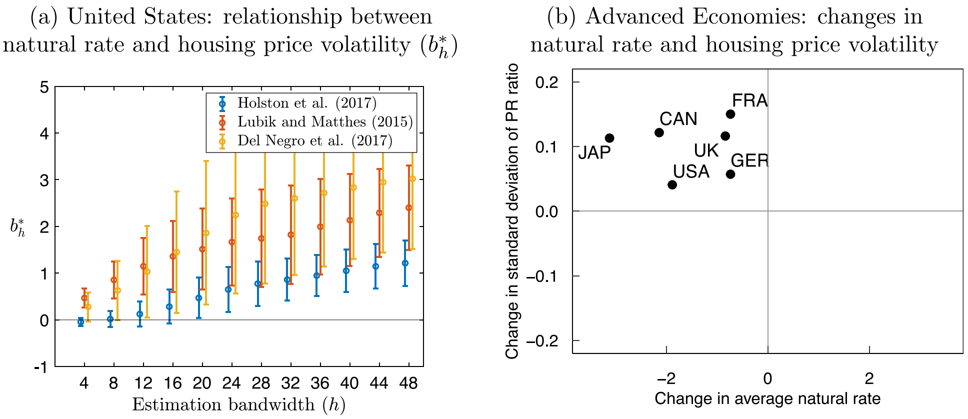

Fig. 3. The natural rate and housing price fluctuations in the data. *Notes*: Panel (a) reports the OLS regression coefficient b_h^* from Eq. (15) for different horizons h together with 68% Newey–West error bands using h quarterly lags. The r_t^* estimates come from Holston et al. (2017), Lubik and Matthes (2015), and Del Negro et al. (2017). The sample periods are 1970Q1 to 2020Q2, except for Lubik and Matthes (2015) which starts in 1967Q1. Panel (b) plots the pre-/post-1990 changes in the Holston et al. (2017) average natural rate, against the changes in the volatility of the price-to-rent ratio for different advanced economies. The sample period is 1970Q1 to 2020Q2. Volatility is again defined as the standard deviation (pre-/post-1990 periods) relative to the period-specific mean value.

Fig. 2(b) depicts the model-implied relationship between the natural rate and the volatility of the price-to-rent ratio. The figure measures volatility by the standard deviation of the price-to-rent ratio divided by its mean, to control for the fact that housing prices increase when the natural rate falls.[18] It shows that lower natural rates are associated with larger housing price volatility, with the effect becoming quite non-linear for low levels of the natural rate.

Next, we document the relationship between natural rates and housing volatility in the data. Let r_t^* denote the long-run level of the natural rate at time t. Under standard assumptions, the long-run level is a function of exogenous fundamentals only.[19] We can thus consider regressions of the form

$$\frac{\mathrm{Std}_h(PR_t)}{\mathrm{Mean}_h(PR_t)} = a_h^* - b_h^* \cdot r_t^* + u_{t,h}, \tag{15}$$

where $\mathrm{Std}_h(PR_t) \equiv \mathrm{Std}(PR_{t-\frac{h}{2}}, \ldots, PR_{t+\frac{h}{2}})$ denotes the standard deviation of the price-to-rent ratio using a window of $h+1$ quarters centered around period t and $\mathrm{Mean}_h(PR_t)$ the average price-to-rent ratio over the same window.

Panel (a) in Fig. 3 reports the coefficients b_h^* for the United States for different estimation bandwidths h. It reports estimates using the long-run natural rate estimates from Holston et al. (2017), which we use as our baseline, but also results using the natural rate estimates of Lubik and Matthes (2015) and Del Negro et al. (2017). For narrow bandwidths, results are sometimes statistically insignificant, which is likely due to the difficulties associated with reliably estimating the standard deviation and the mean of the price-to-rent ratio. For larger bandwidths, however, all coefficients become positive, statistically significant and quantitatively quite large. This shows that the volatility of U.S. housing prices is rising as the long-run natural rate falls.[20]

Panel (b) in Fig. 3 shows that a similar relationship is present in other advanced economies: it plots the change in the average level of the natural rate from the period before 1990 to the period after 1990 for the U.S., Canada, France, Germany, and the United Kingdom, against the change in the volatility of the price-to-rent ratio. In all six advanced economies, the price-to-rent ratio has become more volatile as the average level of the natural rate declined.

Taken together, the structural mechanism in the subjective belief model and the empirical evidence suggest that the volatility of the price-to-rent ratio is positively associated with declines in the average level of the natural rate.

4. Full model with extrapolative housing price beliefs

To analyze the monetary policy implications of falling natural interest rates and rising housing price volatility, we embed subjective beliefs into a general equilibrium sticky price model with a housing sector. All agents are internally rational (see Adam and Marcet, 2011) and maximize utility/profits given their subjective beliefs about housing prices. The policy model is related to the one studied in Adam and Woodford (2021) but features belief distortions that are outside the class of absolutely continuous distortions permitted within their setup. This allows deriving monetary policy implications in a setup featuring quantitatively credible forms of belief distortions. In addition, the present model takes into account the lower-bound constraint on nominal interest rates, which

[18] The volatility measure is thus equal to the standard deviation of the percent deviation of the price-to-rent ratio from its mean.

[19] In the simple model introduced above, the discount rate is the only fundamental. In the full model presented in the next section, the long-run level of the natural rate also depends on long-run productivity growth and preferences.

[20] Since the regressor r_t^* in Eq. (15) is measured with noise, our estimates of b_h^* are conservative, as they suffer from attenuation bias.

we show to be quantitatively important for understanding how the optimal inflation target responds to lower average levels of the natural rate of interest.

4.1. Model setup: households, goods producers, house producers, and the government

We outline the most important model features in this section. A detailed description of the model can be found in Appendix B.

Households. The economy is made up of identical infinitely-lived households that own the good-producing firms as well as the firms in the housing construction sector.[21] Households obtain utility from consumption and from housing and disutility from working, with lifetime utility

$$E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t\right) - \frac{\lambda}{1+\nu} \int_0^1 H_t(j)^{1+\nu} dj + \xi_t^d \left(D_t + D_t^R\right) \right], \tag{16}$$

where C_t is consumption of the final good, $H_t(j)$ is labor supply of type j and $w_t(j)$ the associated real wage. ξ_t^d is the time-varying preference for housing services in the form of owned houses D_t or rented houses D_t^R.

Households optimize subject to the budget constraints

$$C_t + B_t + \left(D_t - (1-\delta)D_{t-1}\right) q_t + R_t D_t^R = \int_0^1 w_t(j) H_t(j) dj + \frac{B_{t-1}(1+i_{t-1})}{\Pi_t} + \frac{\Sigma_t + \Sigma_t^d + T_t}{P_t}. \tag{17}$$

B_t is the real value of government bonds, Π_t is the gross inflation rate, and i_t the nominal interest rate. R_t is the real rental rate for housing. T_t denotes lump-sum taxes and transfers. Profits of firms accrue to households: Σ_t from intermediate goods producers and Σ_t^d from housing constructors.

Households can buy and sell houses at price q_t. Owned houses D_t depreciate at rate δ. Households have subjective beliefs about housing prices but rational expectations about other variables. For tractability, we assume here that agents form beliefs about the housing price in terms of marginal utility units,

$$q_t^u \equiv \frac{q_t}{C_t}, \tag{18}$$

which provides a measure of whether housing is currently expensive or inexpensive in units that are particularly relevant for households.[22] As introduced in Section 3, we assume that households extrapolate housing price growth (now in marginal utility units), $\gamma_t^u \equiv E_t^{\mathcal{P}}\left[\frac{q_{t+1}^u}{q_t^u}\right]$, according to the learning equation

$$\gamma_t^u = \min\left\{ \gamma_{t-1}^u + \frac{1}{\alpha}\left(\frac{q_{t-1}^u}{q_{t-2}^u} - \gamma_{t-1}^u\right), \overline{\gamma}^u \right\}. \tag{19}$$

Households discount future utility at the rate $\beta \in (0,1)$. Since our model is formulated in terms of growth-detrended variables, the discount rate β jointly captures the time preference rate $\widetilde{\beta}$ and the steady-state growth rate of marginal utility. Letting g_c denote the steady-state trend growth rate of consumption, we have

$$\beta = \widetilde{\beta}\frac{1}{(1+g_c)}. \tag{20}$$

When the growth rate g_c of the economy falls, the discount rate β increases. A decline in the trend growth rate of the economy can thus be captured by an increase in the detrended discount factor β. Declining trend growth causes the steady-state natural interest rate to fall, which is in line with the estimates provided in Holston et al. (2017) (see Appendix A.10). Appendix B.1 provides further details on the household optimization problem and the optimality conditions.

Final and intermediate goods producers. A representative final good producer provides an aggregate consumption good

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}. \tag{21}$$

Aggregate inflation Π_t is defined as the price of this final consumption good relative to the price in the previous period and does not include housing services.[23] Each intermediate good i is supplied by a monopolistically competitive producer with a common technology in which (industry-specific) labor $h_t(i)$ is the only variable input:

$$y_{i,t} = A_t f(h_t(i)) = A_t h_t(i)^{1/\phi}, \tag{22}$$

[21] As before, the fact that households are identical is not common knowledge.

[22] Specifying subjective beliefs in units of marginal utility leaves the ability of the learning rule to replicate the survey evidence unchanged. This is because log consumption preferences imply that contributions from fluctuations in marginal utility are orders of magnitude smaller than those generated by subjective beliefs.

[23] Housing prices enter the welfare-relevant inflation measure only in the presence of subjective beliefs, see Section 5, as housing prices are flexible and efficient under rational expectations.

and where A_t denotes exogenous total factor productivity. Intermediate goods producers are subject to a Calvo (1983) price adjustment friction and maximize the value of the firm to the households, using households' subjectively optimal consumption plans to discount profits. The government levies a constant and negative sales tax τ to induce marginal cost pricing in the steady state. Appendix B.2 provides details on the intermediate and final goods producers' optimization problem.

Housing construction firm. A representative housing construction firm operates an isoelastic housing production function to build new houses

$$\tilde{d}_t = \frac{A_t^d}{\tilde{\alpha}} k_t^{\tilde{\alpha}}, \tag{23}$$

where k_t denotes investment into new houses and with $\tilde{\alpha} \in (0,1)$. Appendix B.3 provides further details and optimality conditions.

Government. The government imposes a constant sales tax τ on intermediate goods revenues, issues nominal bonds, and pays lump-sum taxes and transfers T_t to households. The real government budget constraint is given by

$$B_t = B_{t-1} \frac{1 + i_{t-1}}{P_t/P_{t-1}} + \frac{T_t}{P_t} - \tau Y_t.$$

Lump-sum taxes and transfers are set such that they keep real government debt constant at some initial level B_{-1}/P_{-1}. Appendix B.4 provides further details as well as the market clearing conditions.

Equilibrium. The optimality conditions of households, intermediate and final good producers, and housing constructors, the government budget constraints, market clearing conditions, and the nominal interest rate i_t set by the monetary policy authority define an *Internally Rational Expectations Equilibrium* (*IREE*, see Adam and Marcet, 2011), as spelled out in Appendix B.5. To pin down the nominal rate i_t, we will consider Ramsey optimal monetary policy below in Section 5.

4.2. Equilibrium characterization

This section characterizes the equilibrium of the model. To gain analytic insights, we derive a linear–quadratic approximation to the optimal policy problem around the efficient steady state.[24]

Asset prices and housing market equilibrium. From the household optimality conditions, the equilibrium housing price is given by

$$q_t^u = \frac{1}{1 - \beta(1-\delta)\gamma_t^u} \xi_t^d \tag{24}$$

and the price-to-rent ratio by

$$PR_t = \frac{q_t^u}{\xi_t^d}. \tag{25}$$

Variations in subjective beliefs γ_t^u introduce inefficient housing price fluctuations. The economic and welfare implications of these fluctuations can be quantified by the *housing price gap*: $\hat{q}_t^u - \hat{q}_t^{u*}$. The housing price gap measures the log deviation of the actual housing price from its welfare-maximizing level, which in percentage deviations from the steady state is given by

$$\hat{q}_t^{u*} \equiv \frac{1 - \beta(1-\delta)}{1 - \beta(1-\delta)\rho_\xi} \hat{\xi}_t^d. \tag{26}$$

Under rational expectations, the housing price gap is zero at all times, but under subjective beliefs, the housing price gap is given by[25]

$$\hat{q}_t^u - \hat{q}_t^{u*} = \left(\frac{1 - \beta(1-\delta)}{1 - \beta(1-\delta)\gamma_t^u} - \frac{1 - \beta(1-\delta)}{1 - \beta(1-\delta)\rho_\xi} \right) \hat{\xi}_t^d + \frac{\beta(1-\delta)(\gamma_t^u - 1)}{1 - \beta(1-\delta)\gamma_t^u} \tag{27}$$

and depends on the subjective housing beliefs γ_t^u and on the housing preference shocks $\hat{\xi}_t^d$. The housing price gap is of economic interest because it captures misallocations of output between consumption and housing investment. This follows from equation

$$\left((1-\tilde{\alpha})\hat{k}_t - \hat{c}_t \right) - \left((1-\tilde{\alpha})\hat{k}_t^* - \hat{c}_t^* \right) = \hat{q}_t^u - \hat{q}_t^{u*}, \tag{28}$$

in which $\hat{k}_t$ denotes housing investment, $\hat{c}_t$ consumption and starred variables indicate the values with efficient housing prices.[26] The previous equation shows that a positive housing price gap affects the ratio between investment and consumption. Intuitively, this occurs because housing prices affect investment incentives. In particular, high housing prices make housing investment more attractive and lead to a higher investment to consumption ratio. This feature explains why the housing price gap is a welfare-relevant object.

[24] In this steady state, the government levies a negative output tax $\tau = 1/(1-\eta)$ on goods producing firms to eliminate steady-state distortions arising from monopolistic competition.

[25] See Appendix B.9 for the derivation.

[26] All variables are expressed in log deviations from the steady state.

Importantly, Eq. (28) implies that the housing price gap affects consumption even for a *given* level of the output. As a result, the housing price gap will show up in the consumption Euler equation for output, as derived below, because different levels of consumption can be associated with a given path of output, depending on the values assumed by the housing price gap. For the same reasons, the housing gap will affect real wages. The gap will thus also show up in the Phillips curve in addition to output, as we show below.

Finally, we observe that the housing price gap will be more volatile when natural rates of interest are low, because housing prices are more volatile, as discussed in Section 3.4. Detailed derivations underlying the results in this section can be found in Appendix B.9.

Aggregate IS equation and the natural rate of interest. The equilibrium in the goods and housing markets gives rise to an aggregate IS equation. It is given by[27]

$$y_t^{gap} = E_t\left[y_\infty^{gap}\right] - E_t\left[\sum_{k=0}^{\infty}\left(i_{t+k} - \pi_{t+1+k} - \check{r}_{t+k}\right)\right] + \zeta_q\left(\hat{q}_t^u - \hat{q}_t^{u*}\right), \tag{29}$$

and depends on long-run expectations of the output gap, $E_t\left[y_\infty^{gap}\right] \equiv \lim_{T\to\infty} E_t y_T^{gap}$, the path of future real interest rates, and the housing price gap. The variable $\check{r}_t$ denotes the exogenous component of the natural interest rate and depends on the productivity shocks in the economy. The parameter $\zeta_q > 0$ summarizes the aggregate demand effects of housing price gaps. It is the only channel through which subjective housing beliefs influence aggregate demand.

In the special case with efficient housing prices (rational housing price expectations), the IS equation (29) implies that setting $i_t - E_t\pi_{t+1} = \check{r}_t$ for all $t \geq 0$ is consistent with a constant output gap. In the presence of subjective beliefs, however, such a policy fails to keep the output gap stable due to inefficient housing price fluctuations. A policy that sets real interest rates equal to $\check{r}_t$ then delivers

$$y_t^{gap} = E_t\left[y_\infty^{gap}\right] + \zeta_q\left(\hat{q}_t^u - \hat{q}_t^{u*}\right). \tag{30}$$

Since $\zeta_q > 0$, a positive housing price gap is then associated with a positive output gap. As discussed before, high housing prices induce higher housing investment and thereby increase aggregate demand, which leads—for unchanged real interest rates—to an increase in aggregate output. Since the output expansion is inefficient, the policymaker might find it optimal to *lean against housing prices*. The extent to which this is optimal will be explored quantitatively in Section 5 below.

With subjective beliefs, the natural rate of interest, i.e., the real interest rate that is consistent with a constant output gap ($y_t^{gap} = E_t\left[y_\infty^{gap}\right]$ for all t) is given by[28]

$$r_t^* \equiv \check{r}_t - \zeta_q\left(\left(\hat{q}_t^u - \hat{q}_t^{u*}\right) - E_t\left(\hat{q}_{t+1}^u - \hat{q}_{t+1}^{u*}\right)\right) \text{ for all } t. \tag{31}$$

The expression shows that the natural rate under subjective beliefs differs from its value under efficient housing prices if and only if the housing price gap is expected to change in the subsequent period. The natural rate will exceed its level under efficient housing prices, when the expected housing price gap in the next period is higher than the current housing price gap (and vice versa). Since the housing price gap, as well as its first difference, become more volatile as the steady state natural rate falls, Eq. (31) shows how a lower steady state level of the natural rate increases the volatility of the natural rate of interest in the presence of subjective beliefs. This effect is absent with rational housing price expectations.

New Keynesian Phillips curve. The New Keynesian Phillips Curve also depends on housing price gaps. It is given by[29]

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa_y y_t^{gap} + \kappa_q\left(\hat{q}_t^u - \hat{q}_t^{u*}\right). \tag{32}$$

The coefficients $\kappa_y > 0$ and $\kappa_q < 0$ imply that positive output gaps exert positive inflation pressure and positive housing price gaps exert *negative* cost-push effects. This is because inefficiently high housing prices increase housing investment and, for a given output gap, decrease (non-housing) consumption. The latter raises the marginal utility of consumption and thereby depresses wages and the marginal costs for producing the consumption good. In principle, this allows the model to produce a non-inflationary boom in housing prices and housing investment.

4.3. Model calibration and evaluation

We now calibrate the model to explore the quantitative implications for monetary policy of housing price growth extrapolation. The calibration strategy consists of choosing a set of standard parameter values previously considered in the literature and of matching salient features of the behavior of natural interest rates and housing prices in the United States in the pre-1990 period. We then test the model along two dimensions. First, we consider the model predictions for the period 1991–2021 where the average natural rate was significantly lower.[30] Second, we show in Section 4.4 that the model produces reasonable impulse responses to monetary policy disturbances.

[27] See Appendix B.10 for the derivation.

[28] See Appendix B.11 for the derivation.

[29] See Appendix B.12 for the derivation.

[30] We compare across long time spans of 30 years each to obtain more reliable estimates of housing price volatility.

Table 4
Calibration of the full model.

Parameter	Value	Source/Target
Preferences and technology		
β	0.9917	Average U.S. natural rate pre 1990
κ_y	0.057	Adam and Billi (2006)
$\frac{\Lambda_y}{\Lambda_\pi}$	0.007	Adam and Billi (2006)
κ_q	−0.0023	Jointly chosen to match (i) steady state $\frac{k}{c}$ ratio of 6.5%
ζ_q	0.29633	and (ii) long-run housing supply elasticity equal to 5
δ	$\frac{3\%}{4}$	Adam and Woodford (2021)
Exogenous shock processes		
$\rho_{\check{r}}$	0.8	Adam and Billi (2006)
$\sigma_{\check{r}}$	0.1394% [RE: 0.2940%]	Adam and Billi (2006)
ρ_ξ	0.99	Quarterly autocorr. of the price-to-rent ratio of 0.99
σ_ξ	1.65% [RE: 2.33%]	Standard deviation of price-to-rent ratio pre-1990
Subjective belief parameters		
$1/\alpha$	0.7%	Adam et al. (2016)
$\overline{\gamma}^u$	1.0031	Max. percentage deviation of price-to-rent ratio from mean

Calibration to the pre-1990 period. Table 4 summarizes the model parameterization. The quarterly discount factor β is chosen such that the steady-state natural rate equals the pre-1990 average of the U.S. natural rate of 3.34%, as estimated by Holston et al. (2017). The slope of the Phillips curve κ_y, and the welfare weight Λ_y/Λ_π are taken from Adam and Billi (2006, Table 2). The Phillips curve coefficient κ_q and ζ_q are chosen to match the average U.S. housing investment to consumption ratio of 6.5% and the long-run housing supply elasticity of 5, in line with the estimated value in Adam et al. (2012) and in the range of estimates in Topel and Rosen (1988).[31]

As before, we use the AR(1) process (14) for housing demand shocks with a quarterly shock persistence $\rho_\xi = 0.99$. The standard deviation of the innovations to the housing preferences σ_ξ are set such that the model replicates the pre-1990 standard deviation of the price-to-rent ratio. This is achieved by simulating Eqs. (10) and (12), which requires specifying the belief updating parameters α and $\overline{\gamma}^u$. We set $1/\alpha = 0.7\%$ as in Section 3 and determine σ_ξ and $\overline{\gamma}^u$ jointly such that (i) we match the volatility of the price-to-rent ratio and (ii) the simulated data matches the maximum deviation of the price-to-rent ratio in the data from its sample mean. This pins down $\overline{\gamma}^u = 1.0031$ and $\sigma_\xi = 1.65\%$.

We also consider an AR(1) process for the exogenous part of the natural rate

$$\check{r}_t = \rho_{\check{r}}\check{r}_{t-1} + \varepsilon_t^{\check{r}}, \tag{33}$$

where $\varepsilon_t^r \sim iiN(0, \sigma_{\check{r}}^2)$. We set $\rho_{\check{r}}$ as in Adam and Billi (2006) and choose $\sigma_{\check{r}}$ such that the generalized natural rate for the subjective belief model, defined in Eq. (31), also matches the natural rate volatility in Adam and Billi (2006). This yields $\sigma_{\check{r}} = 0.1394\%$.

We also calibrate a rational expectations version of our model. To this end, we recalibrate the volatilities of the innovations to the housing preference and natural rate shock processes to match the volatility of the price-to-rent ratio and the natural rate in the pre-1990 period. This yields $\sigma_\xi = 2.33\%$ and $\sigma_{\check{r}} = 0.2940\%$. Not surprisingly, the RE model requires higher exogenous volatilities to match the empirically observed fluctuations in price-to-rent ratios and natural rates. All other parameters are unchanged relative to the subjective belief model.

Evaluation of the model in the post-1990 period. Fig. 4 illustrates the predictions of the baseline subjective belief model (solid line) for the standard deviation of the price-to-rent ratio (panel (a)) and the standard deviation of the natural rate of interest (panel (b)), conditional on various levels of the steady-state natural rate. The predictions of the RE model are also shown (dashed line). Variations in the steady-state level of the natural rate are achieved via appropriate variations in the discount factor β.[32] The dots in Fig. 4 report the average values for the pre- and post-1990 U.S. sample periods, where the average natural rate was equal to 3.34% and 1.91%, respectively.[33]

Since the model has been calibrated to the pre-1990 period, the model matches the pre-1990 data points in Fig. 4 with subjective beliefs and under rational expectations. The subjective belief model also performs quite well in matching the post-1990 outcomes, despite the fact that these outcomes are untargeted. In particular, the standard deviation of the price-to-rent ratio and the standard deviation of the natural rate endogenously increase as the natural rate falls, with the magnitudes roughly matching the increase observed in the data. In contrast, the RE model produces no increase in the volatility of the natural rate and only a weak increase in the volatility of the price-to-rent ratio. Matching the increase in the natural rate volatility under rational expectations would require increasing $\sigma_{\check{r}}$. We will consider such increases when discussing our quantitative results.

31 See Appendix B.13 for details.

32 As discussed before, variations in the discount factor may be driven by variations in the long-term growth rate and/or by variations in time-preferences.

33 The reported increase in the standard deviation of the natural rate is again based on the estimates in Holston et al. (2017).

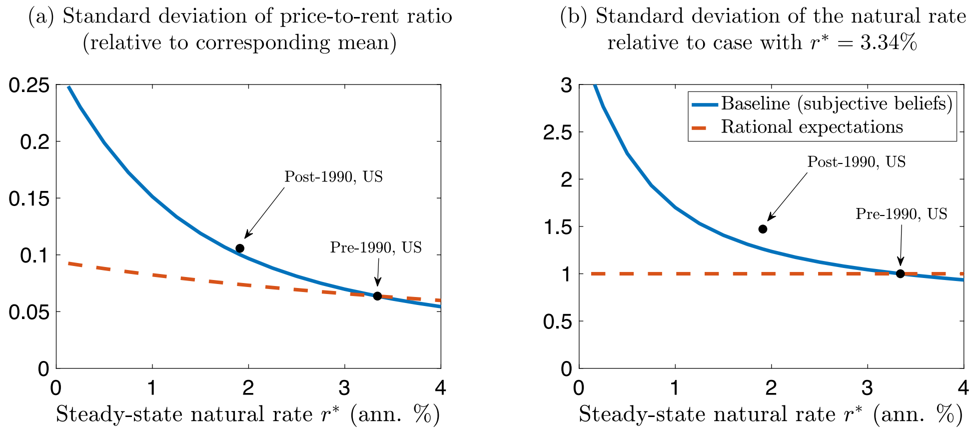

Fig. 4. Standard deviation of price-to-rent ratio and natural rate. *Notes*: This figure plots, for different steady-state levels of the natural rate, the standard deviation of the price-to-rent ratio (relative to its mean) and the standard deviation of the natural rate.

4.4. The effects of monetary policy on housing prices and beliefs

The dynamics of housing prices in marginal utility units, q_t^u, are unaffected by monetary policy, even if housing prices in units of consumption, q_t, do depend on policy. As a result, the object about which agents learn does not depend on policy and the policymaker cannot 'manipulate' households' subjective housing price beliefs in a way to achieve outcomes that are potentially better than under rational expectations.[34] This allows side-stepping the otherwise thorny issue of how the learning rule should respond to the conduct of monetary policy.

Importantly, the model replicates the fact that housing demand and housing investment respond more strongly to monetary policy disturbances than non-housing demand, even though housing prices are flexible and goods price sticky. The model thus avoids the pitfalls of sticky price models with durable goods described in Barsky et al. (2007). Concretely, in response to an exogenous shift in the path of nominal interest rates, $\boldsymbol{i}$, the change in housing investment and consumption satisfies at all times

$$\frac{d \log k_t}{d\boldsymbol{i}} = \frac{1}{1-\tilde{\alpha}} \cdot \frac{d \log C_t}{d\boldsymbol{i}}, \tag{34}$$

where $1/(1-\tilde{\alpha}) > 1$ is the price elasticity of housing supply, see Appendix B.7 for the proof.[35] Appendix B.8 provides a quantitative illustration of how the economy responds to monetary policy, by considering the impulse responses to a monetary policy shock in a simple Taylor rule.

5. Optimal monetary policy in the presence of extrapolative housing price expectations

We now examine how the conduct of optimal monetary policy is affected by subjective housing price beliefs and the average level of the natural rate of interest. To this end, we derive the policymaker's Ramsey problem and show how housing price growth extrapolation generates new monetary policy trade-offs. We focus in the main text on the quantitative implications of housing price beliefs for the optimal inflation target and the policy response to housing demand shocks, but Appendix C.4 characterizes the optimal targeting rule analytically.

5.1. The Ramsey optimal policy problem

We consider optimal monetary policy under commitment with a policymaker that maximizes household utility subject to the constraint that prices and allocations constitute an internally rational expectations equilibrium (IREE). The policymaker holds rational expectations, i.e., understands that the private sectors' housing price beliefs are distorted and thus acts under a probability measure different from the one entertained by households.[36] To obtain analytic insights, we derive the quadratic approximation to the non-linear policy problem[37]:

$$\max_{\{\pi_t, y_t^{gap}, \hat{q}_t^u, i_t \geq \underline{i}\}} -E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left(\Lambda_\pi \pi_t^2 + \Lambda_y \left(y_t^{gap}\right)^2 + \Lambda_q \left(\hat{q}_t^u - \hat{q}_t^{u*}\right)^2 \right) \tag{35}$$

[34] In contrast, Molnar and Santoro (2014), Mele et al. (2020), and Caines and Winkler (2021) consider a setting where policy affects the variables agents use to make inference. Provided agents' learning and forecasting rules are invariant across policy regimes, this allows policy to indirectly influence expectations.

[35] Our calibration uses a supply elasticity of $1/(1-\tilde{\alpha}) = 5$, see Appendix B.13.

[36] Benigno and Paciello (2014) refer to such a policymaker as a "paternalistic" policymaker.

[37] Appendix C.1 presents the non-linear optimal policy problem.

subject to:

$$y_t^{gap} = E_t\left[y_\infty^{gap}\right] - E_t\left[\sum_{k=0}^{\infty}\left(i_{t+k} - \pi_{t+1+k} - \check{r}_{t+k}\right)\right] + \zeta_q\left(\hat{q}_t^u - \hat{q}_t^{u*}\right) \quad (36)$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa_y y_t^{gap} + \kappa_q\left(\hat{q}_t^u - \hat{q}_t^{u*}\right) \quad (37)$$

$$\hat{q}_t^u - \hat{q}_t^{u*} = \left(\frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\gamma_t^u} - \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\rho_\xi}\right)\hat{\xi}_t^d + \frac{\beta(1-\delta)(\gamma_t^u - 1)}{1-\beta(1-\delta)\gamma_t^u} \quad (38)$$

$$\text{Belief updating: } \begin{cases} \gamma_t^u = \min\left\{\gamma_{t-1}^u + \frac{1}{\alpha}\left(\frac{q_{t-1}^u}{q_{t-2}^u} - \gamma_{t-1}^u\right), \overline{\gamma}^u\right\} \\ q_t^u = \frac{1}{1-\beta(1-\delta)\gamma_t^u}\xi_t^d \end{cases} \quad (39)$$

for $t \geq 0$, and initial pre-commitments.

The policymaker's objective (35) is derived in Appendix C.2 and involves the standard terms of squared inflation and the squared output gap, but also depends on the squared housing price gap. The latter arises because deviations of housing prices from their efficient level distort—for a given level of the output gap—housing investment, as explained in Section 4.2. The IS equation (36) and the New Keynesian Phillips Curve (37) are constraints associated with optimal private sector behavior. They both depend on the housing price gap, as discussed in Section 4.2. The housing price gap is determined by Eq. (38) and depends on subjectively expected housing price growth γ_t^u. The dynamics of γ_t^u are jointly determined by the equations in (39).

Since monetary policy cannot directly influence the housing price gap, as explained in Section 4.4, the loss from housing price gaps in the welfare function can be ignored for the computation of optimal policy; housing price gaps thus matter for policy only through their effect on the IS equation and the Phillips curve.

The policymaker's choice of the nominal interest rate i_t is subject to an effective lower bound $i_t \geq \underline{i}$, where the bound $\underline{i} < 0$ is expressed in terms of deviations from the interest rate in a zero-inflation steady state. For the special case with a zero lower bound, we have $\underline{i} = -(1-\beta)/\beta$. In the absence of a lower bound constraint or when economic shocks never cause the bound to become binding, the IS equation (36) can be dropped from the policy problem.

Interestingly, the expectations showing up in the monetary policy problem (35)–(39) are all rational. The way subjective housing price expectations affect the monetary policy problem are thus fully captured through their effects on the housing price gap.

Finally, in the special case with rational housing price expectations, the housing gap is equal to zero at all times, so that the monetary policy problem reduces to the textbook case with a lower bound constraint.

We recursify the optimal policy problem following Marcet and Marimon (2019) and numerically solve for the value functions and optimal policy functions, see Appendix C.3 for details.

5.2. The optimal inflation target

We define the optimal inflation target as the average inflation rate emerging under Ramsey optimal monetary policy. The inflation target thus captures the average inflation rate that the Ramsey planner commits to implement in the stochastic equilibrium. Depending on the realization of shocks, actual inflation will fluctuate around this average value, as discussed below.

Fig. 5(a) depicts the optimal inflation target for different steady-state levels of the natural rate of interest.[38] The figure graphs the optimal target for the setup with subjective housing beliefs (upper solid line), for the case with rational expectations (RE) about housing prices (lower solid line), and for a third case discussed below (dashed line).

Under RE, the optimal target is close to zero for any steady-state level of the natural rate. This confirms earlier findings in Adam and Billi (2006), who considered a high value for the steady-state natural rate and found that the presence of a lower bound constraint cannot justify targeting significantly positive inflation rates. It may be surprising that this holds true also for low steady-state levels of the natural rate: under RE, optimal policy seeks to track the natural rate of interest, but a low steady-state level implies that this is not always feasible. Policy must then use promises of future inflation to lower real interest rates and this more often the lower the steady-state natural rate. Yet, lower-bound episodes are relatively infrequent and short-lived, independently of the steady-state level of the natural rate. As a result, the inflation target does not react strongly to the steady-state (or average) level of the natural rate.

This invariance to the average level of the natural rate differs substantially from the findings in Andrade et al. (2019). They show that the optimal target should move up approximately one-to-one with a fall in the average natural rate under rational expectations. Besides considering a medium-scale sticky price model without housing, a main difference to our approach is that Andrade et al. (2019) study Taylor rules with optimized intercepts rather than optimal monetary policy. Coibion et al. (2012) show that this makes a big difference for how the optimal inflation target responds to lower average values of the natural rate compared to the case with Ramsey optimal policy.

The upper line in Fig. 5(a) shows that the situation is fundamentally different with subjective housing beliefs. The optimal inflation target is overall substantially higher and also depends more strongly on the average natural rate of interest. In particular, a one percentage point decline in the steady-state natural rate increases the optimal inflation target by one third of a percentage

[38] The average level is computed by simulating the model under the optimal monetary policy for 100,000 periods.

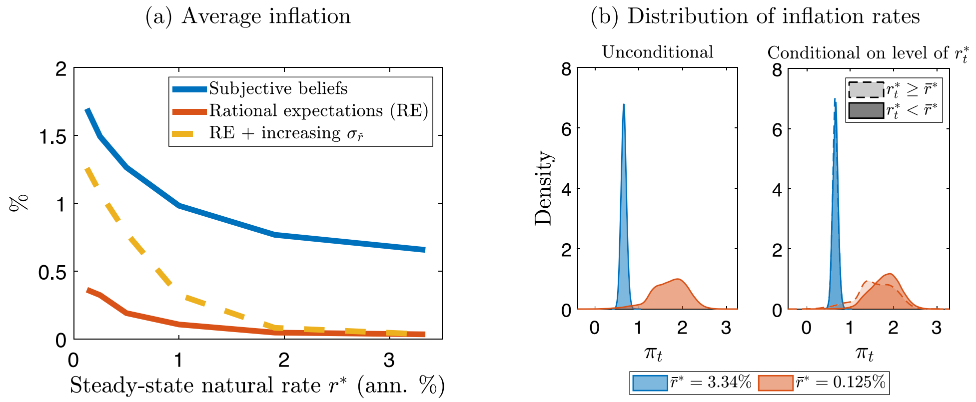

Fig. 5. Average inflation, and distribution of inflation rates, under optimal monetary policy. *Notes*: Panel (a) reports the optimal inflation target for different levels of the steady-state natural rate in the presence of a zero lower bound constraint. The red line shows the optimal target for the case with rational housing price beliefs and the blue line the one with subjective housing price beliefs. The yellow line shows the optimal average inflation under RE where the exogenous volatility of the natural rate is adjusted such that it matches the endogenous volatility increase under subjective beliefs. Panel (b) reports the distribution of inflation rates (in annualized %) for two levels of the steady-state natural rate (also annualized) in the presence of a zero lower bound constraint. The left panel graphs the unconditional distributions of inflation. The right panel conditions distributions on whether the natural rate, r_t^*, is above or below its steady-state level. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

point, with the increase being non-linear and becoming stronger for very low levels of the natural rate. The difference relative to the case with rational expectations arises because the volatility of the natural rate endogenously increases once the natural rate drops, due to the higher volatility of the housing price gaps. This reinforces the stringency of the zero lower bound constraint associated with a lower average value of the natural rate and causes the central bank to engage more often in inflation promises.

The optimal inflation target with subjective housing beliefs is also substantially higher than the optimal target under rational expectations. This holds true even for the pre-1990 average level of the natural rate (3.34%) for which the calibration implies that the natural rate is equally volatile for both belief specifications.

This outcome is due to two reasons: First, fluctuations in the housing price gap also generate cost-push terms in the Phillips curve. Second, belief fluctuations induce more persistent variations in the natural rate than the exogenous natural rate shocks. This puts further upward pressure on the optimal inflation rate, as it requires larger and more persistent inflation promises by the central bank.

To illustrate the role of the *volatility* of the natural rate, the dashed line in Fig. 5 depicts the optimal inflation rate under rational expectations, when we set the volatility of the (exogenous) natural rate in the RE model such that it matches the volatility of the natural rate in the subjective belief model, for each considered level of the natural rate. While the optimal inflation rate increases somewhat relative to the benchmark RE setting, the level of the optimal inflation target still falls short of the one implied by subjective beliefs. However, the optimal inflation target now depends more strongly on the steady-state level of the natural rate and approaches the optimal inflation targets under subjective beliefs when the steady-state natural rate becomes very low.

We can gain further insights into the inflation process under Ramsey optimal policy by considering the stochastic distribution of inflation rates. The left panel in Fig. 5(b) depicts the unconditional distribution of inflation for the lowest and highest steady-state levels of the natural rate considered in Fig. 5(a). It shows that inflation is tightly centered around the optimal inflation target when the steady-state natural rate is high. For a low steady-state natural rate, the inflation distribution shifts to the right, but also displays a long left tail of low inflation rates. This shows that it is optimal for monetary policy to sometimes implement inflation rates that lie significantly below the target.

The right panel in Fig. 5(b) explains why periods of low inflation are optimal: it depicts the inflation distribution conditional on the natural rate being above or below its steady-state value. The left tail with low inflation rates emerges in situations where the natural rate is temporarily high. In such situations, the policymaker does not need to promise future inflation to lower real interest rates, simply because the lower bound constraint is sufficiently far from being binding.[39] This shifts the inflation distribution closer to the one emerging with a high steady-state level for the natural rate. Temporarily high natural rates thus make below-target inflation optimal in a setting where the steady-state natural rate is low. In contrast, it is optimal to induce higher inflation rates—sometimes substantially above the average inflation rate—when the natural rate is below its steady state level. These higher inflation rates capture the promises the monetary authority makes when currently constrained by the lower bound. This differs from a setting with a high steady-state natural rate: conditional distributions are then almost independent of whether the natural rate is above or below its steady-state value, see the right panel in Fig. 5(b).

[39] Due to other state variables shifting around, the lower bound still binds with some probability, but this probability is lower when the natural rate is above its steady-state level.

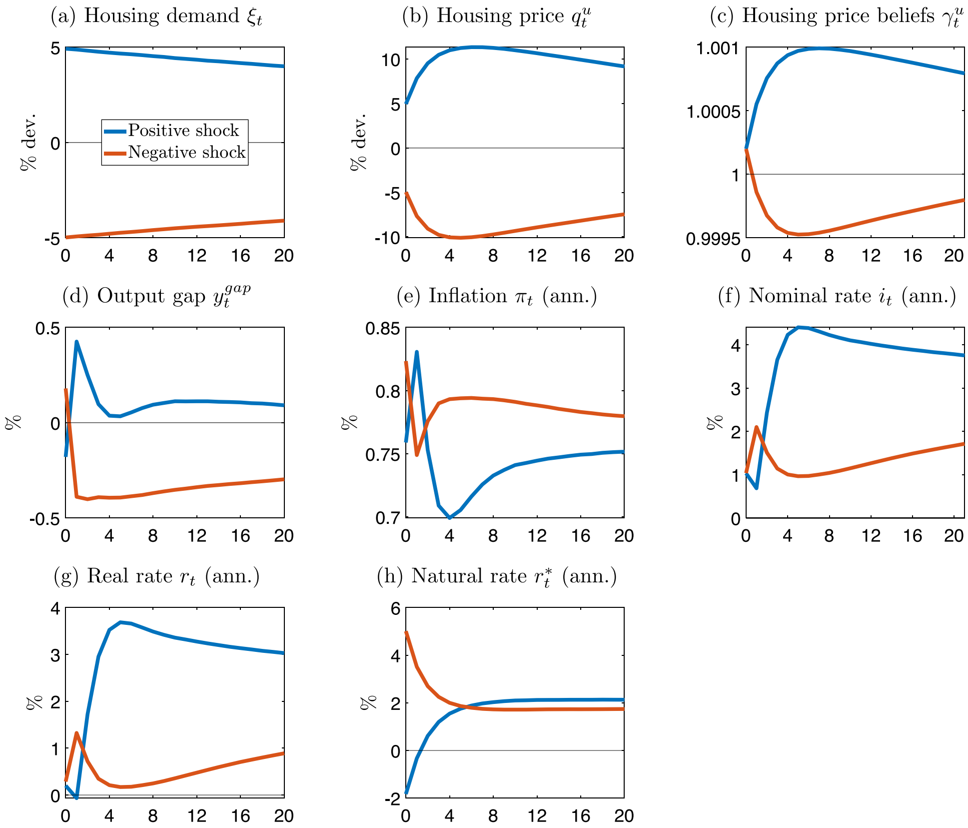

Fig. 6. Impulse responses to a housing preference shock under optimal monetary policy. *Notes*: The figure reports the average impulse responses of the economy under subjective beliefs (at $r^* = 1.91\%$) after a three-standard-deviation housing demand shock. The economy is initialized at its ergodic mean. The impulse responses are obtained by averaging the subsequent response over the possible future shock realizations. The blue lines show the responses after a positive shock and the red lines after a negative shock.

5.3. *Asymmetric leaning against housing demand shocks*

We now examine the optimal monetary policy response to housing demand shocks. Under RE, housing demand shocks do not affect the housing price gap and thus also do not generate policy trade-offs. This is different with subjective housing price beliefs. Housing demand shocks then move housing prices and thereby housing price expectations. The latter then affect the housing price gap, which generates movements in the natural rate and leads to cost-push terms.

We show that this makes it optimal to 'lean against' housing demand shocks in the presence of subjective beliefs. Due to the lower bound constraint, however, the responses to positive and negative housing demand shocks display considerable asymmetry: optimal monetary policy considerably tightens following shocks that give rise to housing price booms but only weakly stimulates following shocks producing a housing price bust. This is so because stimulative reductions in current interest rates are constrained by the lower bound, so that stimulation works partly by promising higher inflation in the future. This raises average inflation in a situation where inflation is (on average) high already, which is costly in welfare terms. Restrictive policy, in contrast, decreases inflation thus has additional benefits in terms of bringing inflation closer to the welfare optimal target.

The top row in Fig. 6 shows the response of housing-related variables to a persistent positive/negative housing demand shock of three standard deviations.[40] On impact, the shock triggers housing price growth of about 5%, which then triggers belief revisions that fuel further movements of the housing price in the same direction. The positive shock, for instance, pushes housing prices up by about 5% on impact, with belief momentum generating approximately another 5% in the first six quarters after the shock. This causes the housing price gap to become significantly positive (not shown in the figure). Once actual housing price increases start to fall short of the expected housing price increases, the housing boom reverts direction. With the housing price gaps entering positively in the IS equation and *negatively* as a cost-push term in the NKPC, a positive (negative) housing demand shock exerts upward (downward)

[40] We initialize the economy at its ergodic mean – where the economy is at its average inflation rate, interest rate, and output gap – and then hit the economy with a one-time shock of three standard deviations in period 0. We then simulate 100,000 paths of the economy with random shocks for the subsequent, and thereby average over the possible future shock realizations after the initial shock. We assume a steady-state natural rate equal to its post-1990 mean (1.91%).

pressure on the output gap and negative (positive) cost-push effects. As explained in Section 4.2, cost-push effects arise because inefficiently high housing prices increase housing investment and, for a given output gap, decrease (non-housing) consumption. The latter depresses wages and the marginal costs for producing the consumption good.

Higher housing prices push up housing investment, which causes upward pressure on the output gap. Optimal monetary policy leans strongly against the housing price and increases nominal and real interest rates. It does so despite the fact that the natural rate of interest falls in response to the shock. The policy response causes a fall in inflation, which is amplified by the fact that the increase in housing prices and investment increases the marginal utility of consumption, hence, dampens wages and marginal costs. A positive housing demand shock thus results—in the presence of subjective housing beliefs—in a disinflationary housing boom episode under optimal monetary policy.

The policy response to a positive housing demand shock is much stronger than that to a negative housing demand shock. In particular, nominal and real interest rates fall considerably less following a negative shock realization. This is so because a negative housing price gap is inflationary and inflation is already high to start with. Negative housing demand shocks thus move inflation further away from its optimal level of zero.[41] Yet, policy still 'leans against' the housing price decrease: real interest rates fall despite the fact that the natural rate increases. Indeed, the asymmetric strength of the policy response is largely due to the initial level of inflation: even absent of cost-push effects of housing price gaps, the monetary policy response is stronger in response to positive housing demand shocks, as shown in Appendix C.5. To some extent, however, the absence of cost-push effects gives policy more room to let inflation increase, and thus makes the optimal policy response to negative housing demand shocks stronger.

The fact that leaning against housing prices can be optimal in the presence of housing price growth extrapolation is in line with results in Caines and Winkler (2021), who consider a setting with "conditionally model consistent beliefs" in which expectations about many variables differ from rational expectations, and with results in Adam and Woodford (2021), who consider a setting where the policymaker fears "worst-case belief distortions" about inflation and housing price expectations. As none of these papers consider a lower-bound constraint, the policy response to positive and negative shocks is symmetric in their settings.

In a model extension, we analyze whether macroprudential policies could be employed to address the housing market inefficiencies generated by the presence of subjective price expectations. However, we find that taxes on housing that are dynamically adjusted so as to reduce housing price fluctuations would have to be large and very volatile. Online Appendix D provides details.

6. Conclusion

This paper empirically documents three key deviations of households' housing price expectations from full-information rational housing price expectations and constructs a structural equilibrium model that jointly replicates the behavior of housing prices and the patterns of subjective beliefs. The model shows that subjective housing price beliefs significantly contribute to housing price fluctuations and that lower natural rates of interest generate increased volatility for housing prices and the natural rate.

Optimal monetary policy responds to lower and more volatile natural rates by implementing higher average inflation rates. Monetary policy should also lean against housing price fluctuations induced by housing demand shocks, with reactions to housing price increases being more forceful than the reaction to housing price downturns. None of these features is optimal if households hold rational housing price expectations. This highlights the importance of basing policy advice on economic models featuring empirically plausible specifications for household beliefs.

Data availability

Data will be made available on request.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jmoneco.2024.103647.

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[41] While the output gap is moved closer to its optimal level, the weight on the output gap in the welfare function is two orders so magnitude smaller than that on inflation, see Table 4.

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